INSTRUCTIONS

Each question is printed both in English and in Kannada.

Answers must be written in the medium specified (English or Kannada) in the Admission Ticket issued to you, which must be stated clearly on the cover of the answer-book in the space provided for this purpose. No credit will be given for the answers written in a medium other than that specified in the Admission Ticket.

Answer any five questions.

All questions carry equal marks.
1. (a) Is the vector $\alpha = (2, -5, 3)$ in $V_3(\mathbb{R})$ a linear combination of vectors $\alpha_1 = (1, -3, 2), \alpha_2 = (2, -4, -1)$ and $\alpha_3 = (1, -5, 7)$?

(b) Set up the linear transformation $Y = AX$ which carries $E_1$ into $Y_1 = (1, 2, 3)^T$ and $E_3$ into $Y_3 = (2, 1, 3)^T$. Also find the images of $X_1 = (1, 1, 1)^T$, $X_2 = (3, -1, 4)^T$ and $X_3 = (4, 0, 5)^T$.

(c) Given $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, compute $A^{-2}$ and $A^4$.

2. (a) Reduce the symmetric matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}$ to canonical form.

(b) Prove that the relation of similarity is an equivalence relation in the set of all $n \times n$ matrices over a field $F$.

(c) Let $A$ be an $m \times n$ matrix with real entries. Prove that $A = 0$ (null matrix) if and only if the trace $(A^T A) = 0$.

3. (a) Define positive definite and positive semi-definite matrix. Show that

$$\overline{X}^T \begin{bmatrix} 1 & 1+i & -1 \\ 1-i & 6 & -3+i \\ -1 & 3-i & 11 \end{bmatrix} X$$

is positive definite.

(b) Determine the characteristic roots and associated invariant vectors of

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

(c) Show that the Hermitian matrix $A = \begin{bmatrix} 1 & 1+i & 2+3i \\ 1-i & 2 & -i \\ 2-3i & i & 0 \end{bmatrix}$ can be written as $B + iC$, where $B$ is real and symmetric and $C$ is real and skew symmetric.
1. (a) $V = (IR) \in \mathbb{C}, \alpha_1 = (1, -3, 2), \alpha_2 = (2, -4, 1), \alpha_3 = (1, -5, 7)$

(b) \( Y = (1, 2, 3)^T \in \mathbb{C}, E_1 = (1, 2, 3)^T \in \mathbb{C}, E_2 = (1, 3, 4)^T \in \mathbb{C} \)

2. (a) \( A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \) and \( A^{-2} = A^2 \) is also found.

3. (a) $\mathbb{F}_{2^3} [x] \in \mathbb{C}_3, \mathbb{F}_{2^3} [x] \in \mathbb{C}_3$, \( m \times n \) matrices are considered. For matrix \( A \), \( \text{det}(A) = 0 \) and matrix is not invertible.

(b) $F \in \mathbb{F}_{2^3}$, \( n \times n \) matrices are considered. \( \text{det}(A) = 0 \) and matrix is not invertible.

(c) $A$ is a matrix for which the determinant is zero. \( \text{det}(A^T A) = 0 \) and matrix is not invertible. $A = 0$ (null matrix).

3. (a) \( \overline{X}^T \begin{bmatrix} 1 & 1+i & -1 \\ 1-i & 6 & 3+i \\ -1 & 3-i & 11 \end{bmatrix} \overline{X} \neq \mathbf{0} \) for all $\overline{X}$.

(b) \( A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \) and any submatrix is non-singular.

(c) $B$ is a matrix and $C$ is a matrix. \( B + iC \) is considered.

\[ A = \begin{bmatrix} 1 & 1+i & 2+3i \\ -i & 2 & -i \\ -2-3i & i & 0 \end{bmatrix} \]

[Turn over]
4. (a) Gas is escaping from a spherical balloon at the rate of 2 cubic cm per sec. How fast is the surface area shrinking when the radius is 16 cm?

(b) Trace the curve \( a^2 y^2 = x^2 \left( a^2 - x^2 \right) \).

(c) Find the area bounded by the curve \( \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^{2/3} = 1 \).

5. (a) Find the asymptotes of the curve \( x^2 y^2 - x^2 y - 2xy^2 + x + y + 1 = 0 \).

(b) Prove that \( \beta (m, n) = \frac{\Gamma (m) \Gamma (n)}{\Gamma (m + n)} \).

(c) Using double integration, find the centre of gravity of a lamina in the shape of a quadrant of the curve \( \left( \frac{x}{a} \right)^{2/3} + \left( \frac{y}{b} \right)^{2/3} = 1 \), the density being \( \rho = kxy \), where \( k \) is a constant.

6. (a) Find the equation of the parabola with focus at \((1, -1)\) and with \(3x - 4y = 2\) as its directrix. Also find the latus rectum.

(b) Find the equation of the pair of lines through the origin perpendicular to the lines \(ax^2 + 2hxy + by^2 = 0\).

(c) A point moves so that its distance from the point \((4, 0)\) is always one half its distance from the line \(x - 16 = 0\). Show that the equation of its locus is an ellipse.
4. (a) ನೋಡಿ ಈತರಾಧ್ಯಗಳ ಮಾಡಿಸಿದ್ದಾರೆ ತಂತ್ರಣ 2 ತಿ.ಮ. ಸೇರಿಸಿ, ಕಂಡು ಪಡೆಯಬಲ್ಲದೇ. ತನ್ನಿಗೆ 16 ತಿ.ಮ. ಅಂತಕಾಲ ಮತ್ತು ಸೇರಿಸಿದ ತಂತ್ರಣ ಎಂದರೆ ಎಂಬುದು ಎಂದರೆ, ಎಂಬುದು ಎಂಬುದು?

(b) \( a^2 y^2 = x^2 \left( a^2 - x^2 \right) \) ಎಂಬರು ಎಂಬರು.

(c) \( \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^{2/3} = 1 \) ನೋಡಿ ಎಂಬರು ಎಂಬರು. ಎಂಬರು ಎಂಬರು.

5. (a) \( x^2 y^2 - x^2 y - 2xy^2 + x + y + 1 = 0 \) ಎಂಬರು ಎಂಬರು ಎಂಬರು ಎಂಬರು.

(b) \( \beta (m, n) = \frac{\Gamma (m) \Gamma (n)}{\Gamma (m + n)} \) ಎಂಬರು.

(c) \( k \) ಎಂಬರು ಎಂಬರು ಎಂಬರು, \( p = kxy \) ಎಂಬರು. \( \left( \frac{x}{a} \right)^{2/3} + \left( \frac{y}{b} \right)^{2/3} = 1 \) ಎಂಬರು ಎಂಬರು. ಎಂಬರು ಎಂಬರು ಎಂಬರು ಎಂಬರು ಎಂಬರು,

6. (a) \( (1, -1) \) ಎಂಬರು ಎಂಬರು ಎಂಬರು ಎಂಬರು 3x - 4y = 2 ಎಂಬರು. ಎಂಬರು ಎಂಬರು. ಎಂಬರು ಎಂಬರು.

(b) \( ax^2 + 2hxy + by^2 = 0 \) ಎಂಬರು. \( \alpha \) ಎಂಬರು. \( \beta \) ಎಂಬರು. \( \gamma \) ಎಂಬರು. \( \delta \) ಎಂಬರು.

(c) \( \beta \) ಎಂಬರು. \( (4, 0) \) ಎಂಬರು. \( \beta \) ಎಂಬರು. \( x - 16 = 0 \) ಎಂಬರು. \( \beta \) ಎಂಬರು. \( \alpha \) ಎಂಬರು. \( \delta \) ಎಂಬರು. \( \gamma \) ಎಂಬರು. \( \delta \) ಎಂಬರು.
7. (a) Find the equation of the two spheres which touch the z-axis and pass through the circle $x^2 + y^2 - 4x - 6y + 1 = 0$. 

(b) Find the equations in symmetric form for the line of intersection of the planes $x - y + 3z = 2$ and $3x + y + z = 2$. 

(c) Find the equation of the sphere having its centre on the plane $4x - 5y - z = 3$ and passing through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0$. 

$x - 2y + z = 8$. 

8. (a) Solve $e^y \cos x \, dy + \left( e^y + 1 \right) \sin x \, dx = 0$. 

(b) Solve $y'' = x + xy'$, $y(0) = 1$, $y'(0) = 0$. 

(c) A boat is rowed with a velocity $\mu$ directly across a stream of width $\alpha$. If the velocity of the current is directly proportional to the product of the distances from the two banks, find the path of the boat and the distance downstream to the point where it lands. 

9. (a) Solve $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ 

(b) Solve $\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2}$. 

(c) Solve $(3x + 2)^2 y'' + 5(3x + 2) y' - 3y = x^2 + x + 1$. 


7. (a) \( z = x^2 + y^2 - 4x - 6y + 1 \), \( z = 0 \) is a plane tangent to the surface \( x^2 + y^2 = 4z + 6y + 1 \) at point \( (x, y, z) \).

(b) \( x - y + 3z = 2 \) \( \Rightarrow \) \( 3x + y + z = 2 \) \( \Rightarrow \) \( x - y + 3z = 2 \) is a plane \( \Rightarrow \) \( x - y + 3z = 2 \) is a plane tangent to the surface \( x^2 + y^2 = 4z + 6y + 1 \) at point \( (x, y, z) \).

(c) \( 4x - 5y - z = 3 \) \( \Rightarrow \) \( x^2 + y^2 = 2x - 3y + 4z + 8 = 0 \), \( x - 2y + z = 8 \) \( \Rightarrow \) \( x^2 + y^2 = 2x - 3y + 4z + 8 = 0 \) is a plane tangent to the surface \( x^2 + y^2 = 2x - 3y + 4z + 8 = 0 \) at point \( (x, y, z) \).

8. (a) \( \frac{dy}{dx} \cos x \, dy + \left( e^y + 1 \right) \sin x \, dx = 0 \).

(b) \( \frac{dy}{dx} = x + xy' \), \( y(0) = 1 \), \( y'(0) = 0 \).

(c) \( \mu \text{ damped mass } m \text{ spring } \Rightarrow \) \( a \text{ spring } \Rightarrow \) \( \text{ damping } \Rightarrow \) \( \text{ natural frequency} \), \( \mu \text{ damping } \Rightarrow \) \( \text{ mass } \Rightarrow \) \( \text{ spring } \Rightarrow \) \( \text{ damping} \), \( \mu \text{ damping } \Rightarrow \) \( \text{ mass } \Rightarrow \) \( \text{ spring } \Rightarrow \) \( \text{ damping} \).

9. (a) \( x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4 \).

(b) \( \frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2} \).

(c) \( (3x + 2)^2 y'' + 5(3x + 2) y' - 3y = x^2 + x + 1 \)
10. a) Constant forces \( \mathbf{F} = 2 \mathbf{i} - 5 \mathbf{j} + 6 \mathbf{k} \) and \( \mathbf{G} = -\mathbf{i} + 2 \mathbf{j} - \mathbf{k} \) act on a particle. Determine the work done when the particle is displaced from \( A \) to \( B \), the position vectors of \( A \) and \( B \) being \( 4 \mathbf{i} - 3 \mathbf{j} - 2 \mathbf{k} \) and \( 6 \mathbf{i} + \mathbf{j} - 3 \mathbf{k} \) respectively.

(b) Apply Stoke's theorem to prove that
\[
\oint_C (y \, dx + z \, dy + x \, dz) = -2\sqrt{2} \pi a^2 ,
\]
where, \( C \) is the curve given by
\[
x^2 + y^2 + z^2 - 2ax - 2ay = 0, \quad x + y = 2a
\]
and begins at the point \((2a, 0, 0)\) and goes at first below the \( z \)-plane.

(c) Prove that the direct product of the two tensors result in a new tensor of rank equal to the sum of ranks of there tensors.

11. (a) Three forces \( P, Q, R \) act along the sides of the triangle formed by the lines \( x + y = 1, \ y - x = 1 \) and \( y = 2 \). Find the equation to the line of action of the resultant.

(b) A string of length \( a \) forms the shorter diagonal of a rhombus formed of four uniform rods each of length \( b \) and weight \( W \), which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is
\[
\frac{2W \left( 2b^2 - a^2 \right)}{b \sqrt{4b^2 - a^2}} .
\]

(c) A particle of mass \( m \) is falling under gravity in a medium whose resistance equals \( \mu \) times the velocity. If the particle be released from rest, find the distance described in time \( t \).
10. (a) \[ P = 2\mathbf{T} - 5\mathbf{J} + 6\mathbf{k} \quad \text{and} \quad Q = -\mathbf{T} + 2\mathbf{J} - \mathbf{k} \]

(b) \[ x^2 + y^2 + z^2 - 2ax - 2ay = 0, \quad x + y = 2a \text{ given} \quad \int_{C} (y \, dx + z \, dy + x \, dz) = -2\sqrt{2} \pi a^2 \]

(c) \[ \text{ secured ranks and non-secured ranks} \]

11. (a) \[ x + y = 1, \quad y - x = 1 \text{ and} \quad y = 2 \text{ given} \quad \text{to be solved for} \ P, \ Q, \ R \]

(b) \[ \frac{2W}{b} \left( \frac{2b^2 - a^2}{b \sqrt{4b^2 - a^2}} \right) \]

(c) \[ m \text{ given,}\quad \text{to be solved for} \]

| Turn over |
12. (a) A person falls by means of a parachute from a height of 900 metres in 150 seconds. Taking the resistance to vary as the square of the velocity, determine the limiting velocity.

(b) Prove that the horizontal line through the centre of pressure of rectangle immersed in a liquid with one side in the surface divides the rectangle into two parts, the fluid pressures on which are in the ratio 4 : 5.

(c) A conical glass is filled with water and placed in an inverted position upon a table. Show that the resultant vertical thrust of the water on the glass is two-thirds that on the table.
12. (a) ಕ್ಯಾರ್ನಿವಾಲ್ ಪ್ರೋಪ್ ಸಿಸ್ಟೆಮ್ 900 ಮೇಟ್ಟಿಗೆ ಇತ್ಯಾದಿಯಾಗಿ 150 ಪೋಸಿಷನ್ ಎಂಬುದು. ವರ್ಗಾಕ್ರದ ಪ್ರೋಪ್ ಸಿಸ್ಟೆಮ್ನ ಪ್ರಧಾನವಾಗಿ ಬೆಂಬಲದಲ್ಲಿ ಸಂಖ್ಯೆಯ ಉಚ್ಛಾರಣೆಯ ಅಂಕಗಳನ್ನು ಪಡೆದುಕೊಂಡಿರಲಿಲ್ಲ.

(b) ಅನೇಕ ಕ್ಯಾರ್ನಿವಾಲ್ ಪ್ರೋಪ್ ಸಿಸ್ಟೆಮ್ ಪ್ರವರಿ ಸಿಮಿಟಲ್ ಸೂತ್ರದ ಮೂಲಕ ಗೂಡಿಸಿರುವುದು, ಮೂಲಕು ಪ್ರಧಾನವಾಗಿ ಸಿದ್ದಿಸುವ ಪ್ರಕಾರದಲ್ಲೇ ವಿಭಾಗ ನಡೆದು ಉಪಕ್ರಮದ ಅಂಕಗಳ ಸರಳತೆಯನ್ನು 4 : 5 ಎಂಬುದು.

(c) ಮೃದು ಸಂಕೀರ್ಣವಾಗಿ ಕೋರಿ ಹೊಂದಿದ್ದು ಇದು ಪ್ರಧಾನವಾಗಿ ಸಂಖ್ಯೆಯ ಮಾಹಿತಿಯ ಉಪಕ್ರಮದ ಕುಂಠಿಯ (Inverted position) ಮೂಲಕಿರುವುದು. ಇದರ ಆಧಾರದಲ್ಲಿ ಸುಮಾರು ವಿಭಾಗದ ಸಂಖ್ಯೆಯ ವಿಶೇಷ ಸೂಚನೆಯನ್ನು (Thrust) ಮೂಲಕಿರುವುದು ಸಂಖ್ಯೆಯ ವಿಭಾಗದ ಸಂಖ್ಯೆಯಿಂದೇ ಮೃದುವಿದ್ದು.
INSTRUCTIONS

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Answer any five questions, choosing at least two from each Section.

All questions carry equal marks.
SECTION - A

1. (a) Show that the four fourth roots of unity form a multiplicative group.

(b) Prove that any sub-group of a cyclic group is cyclic.

(c) If $G$ is a finite Abelian group and a positive integer $k$ divides $o(G)$, prove that $G$ contains a subgroup of order $k$.

2. (a) If in a ring $R$ with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$, prove that $R$ is commutative.

(b) Show that for any field $F$, the ring of polynomials $F[x]$ is a Principal Ideal Domain (PID).

(c) Let $R$ be a Euclidean ring. Let $a$ and $b$ be two non-zero elements in $R$. Prove that if $b$ is not a unit in $R$, then $d(ab) > d(a)$.

3. (a) Test the convergence of

$$\frac{1}{\log 2} + \frac{1}{\log 3} + \ldots + \frac{1}{\log n} + \ldots$$

(b) Prove that $f$ is continuous on a metric space $S$ if and only if $f$ is continuous on every compact subset of $S$.

(c) Evaluate the integral

$$\iint xy \, dx \, dy$$

over the positive quadrant of the circle $x^2 + y^2 = a^2$.

4. (a) (i) Check the uniform convergence of $\tan x$ in $[0, \pi/2]$.

(ii) Find the maxima of $y = x^3 - 7x^2 + 2$ in $[0, 5]$.

(b) If $f \in R[a, b]$, then prove that $\int_a^b f = -\int_b^a f$.

(c) Examine the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$.

5. (a) Extend Cauchy's theorem concerned with vanishing of integrals along closed curves to a multiply connected region (region with holes where the function is not defined).

(b) Find the Taylor's series to represent $\frac{z^2}{(z+2)(z+3)}$ about 0.

(c) Prove that if a power series in $z$ is convergent at $z = z_1$, then it converges absolutely in $|z| < |z_1|$.
1. (a) $\mathbb{R}^n$ (Unity) $\subseteq \mathbb{R}^n$, $\mathbb{R}^n$ is a vector space under the operations of vector addition and scalar multiplication.

(b) $\mathbb{R}$ is a field. $\mathbb{R}$ is a commutative ring with unity (Commutative) under addition and multiplication.

(c) $G$ is a group. $G$ is a monoid under addition. $k \in G$. $o(G)$ is the order of $G$. $G$ is a group under addition.

2. (a) $x, y \in \mathbb{R}$, $R$ is a field, $\mathbb{R}$ is a commutative field $\mathbb{R}$. $(xy)^2 = x^2 y^2$ holds, $\mathbb{R}$ is a field (Commutative) under addition and multiplication.

(b) $\mathbb{Z}$ is a field, $\mathbb{Z}$ is a field. $\mathbb{Z} / \mathbb{Z}$ is a field (PID) under addition and multiplication.

(c) $R$ is a ring. $R$ is a ring. $a, b \in R$. $a + b = b + a$ holds. $R$ is a commutative ring.

$3. (a) \frac{1}{\log 2} + \frac{1}{\log 3} + \ldots + \frac{1}{\log n} \leq \sum_{k=2}^{n} \frac{1}{k}$

(b) $S$ is a subspace of $\mathbb{R}^n$ (Compact) under addition and scalar multiplication.

(c) $x^2 + y^2 = a^2$ is the equation of a circle.

$\int xy \, dx \, dy$ is the area of the circle.

4. (a) (i) $\left[ 0, \frac{\pi}{2} \right]$ $\subset \mathbb{R}$, $\tan x$ is continuous on $\left[ 0, \frac{\pi}{2} \right]$.

(ii) $\left[ 0, 5 \right]$ $\subset \mathbb{R}$, $y = x^3 - 7x^2 + 2$ is continuous on $\left[ 0, 5 \right]$ (Maxima) exists.

(b) $f \in \mathbb{R} \setminus \{a, b\}$ holds, $\int_a^b f \, dx$ is continuous.

(c) $\int_0^1 \frac{dx}{x^{1/2} \left( 1 - x \right)^{1/3}}$ is continuous on $\left( 0, 1 \right)$.

5. (a) $\lim_{x \to a} f(x) = L$ (Limit) $\subseteq \mathbb{R}$, $\lim_{x \to a} f(x) = L$ holds.

(b) $\lim_{x \to a} \frac{z^2 - 1}{(z + 2)(z + 3)}$ is continuous. $\lim_{x \to a} f(x) = L$ holds.

(c) $x = z_1 + z_2$, $\lvert z \rvert < \lvert z_1 \rvert$. $\sum_{n=0}^{\infty} a_n z^n$ (Power series) $\sum_{n=0}^{\infty} a_n z^n$ (Convergent) $\sum_{n=0}^{\infty} a_n z^n$ (Convergent) $\sum_{n=0}^{\infty} a_n z^n$ (Convergent).
6. (a) Evaluate \( \int_C \frac{z^2 + 1}{z^2 - 4} \, dz \) if \( C \) is the circle of unit radius with centre at

(i) \( z = 2 \), (ii) \( z = -2 \) and (iii) \( z = 0 \).

(b) State and prove Cauchy's residue theorem.

(c) Prove that \( \int_0^\infty e^{-x^2} \cos^2 \alpha \sin \left( x^2 \sin 2\alpha \right) \, dx = \frac{\sqrt{\pi}}{2} \sin \alpha \).

7. (a) Find the differential equation of all planes which are at a constant distance from the origin.

(b) Solve: \( \left( x^2 - y^2 - z^2 \right) p + 2xyq = 2xz \).

(c) Use Charpit's method to solve \( \left( p^2 + q^2 \right) y = qz \).

8. (a) Solve: \( p^2 + q^2 = z^2 \left( x^2 + y^2 \right) \).

(b) Solve: \( \frac{\partial^3 Z}{\partial x^3} - 2 \frac{\partial^3 Z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y \).

(c) Solve:

(i) \( p^3 + q^3 = 27z \)

(ii) \( yp = 2xy + \log q \).
6. (a) (i) \( z = 2, \) (ii) \( z = -2 \) (iii) \( z = 0 \) तीनोऽऽ मागळातील \( C \) व नात \( z = \alpha \) \( T \) इलेक्ट्रिक \( \int_C \frac{z^2 + 1}{z^2 - 4} \, dz \) वर्गमयी गरिते देश.

(b) \( \text{Cauchy's} \) उदाहरण \( \int_{C} \frac{1}{z^4 - 4} \, dz \) वर्गमयी आबोध.

(c) \( \int_0^\infty e^{-x^2} \cos^2 \alpha \sin \left( x^2 \sin 2\alpha \right) \, dx = \frac{\sqrt{\pi}}{2} \sin \alpha \) \( \text{वर्गमयी} \) आबोध.

7. (a) \( \text{Origin} \) व नात \( \alpha \) राष्ट्र मागळातील \( \alpha \) \( \text{वर्गमयी} \) आबोध \( \text{वर्गमयी} \) नतींगी देश.

(b) \( p \) \( \text{वर्गमयी} \) \( \left( x^2 - y^2 - z^2 \right) p + 2xyq = 2xz. \)

(c) \( \left( p^2 + q^2 \right) y = qz \) \( \text{वर्गमयी} \) आबोध \( \text{वर्गमयी} \) देश.

8. (a) \( \text{वर्गमयी} \) \( p^2 + q^2 = z^2 \left( x^2 + y^2 \right). \)

(b) \( \text{वर्गमयी} \) \( \frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y. \)

(c) \( \text{वर्गमयी} \)

(i) \( p^3 + q^3 = 27z \)

(ii) \( yp = 2xy + \log q. \)
9. (a) Using Lagrange's equations obtain the differential equations for planetary motion.

(b) Calculate the moment of inertia of a uniform elliptic lamina about (i) the tangent at the extremity of the major axis and (ii) the axis through the centre of the lamina and perpendicular to the plane.

(c) A pulley system is given as shown in the diagram. Discuss the motion of the system using Lagrange's method when the pulley wheels have negligible masses and moments of inertia and their wheels are frictionless.

10. (a) Show that the motion of a particle of mass \( m \) projected vertically upwards with a velocity \( u \) in a medium whose resistance varies as the square of the speed is governed by

\[
\log \left( \frac{V^2 + u^2}{V^2 + u'^2} \right) = \frac{2g}{V^2} x,
\]

where \( x \) is the coordinate in which the motion takes place, \( V = \sqrt{\frac{g}{\mu}} \) is the terminal velocity and \( \mu u^2 \) is the magnitude of resistance per unit mass.
9. (a) Planetary motion  

(b) (i) 

(ii) 

(c) 

10. (a) \[ V = \sqrt{\frac{g}{\mu}} \]  

\[ \log \left( \frac{V^2 + u^2}{V^2 + u^2} \right) = \frac{2g}{V^2} \]  

m \[ \text{ where } \mu \text{ is the mass of the central body and } m \text{ is the mass of the object.} \]
(b) Investigate the motion of a particle moving under an attractive inverse square law. Given that \( \frac{m h^2}{p^2} \frac{dp}{dn} = -F \) is the pedal equation of the orbit under a central force \( (\text{where } h = r^2 \theta ) \).

(c) Discuss D'Alembert's Principle.

11. (a) Give an example for two-dimensional potential flow. Discuss how the stream function satisfying Laplace equation that arises in Hydrodynamics.

(b) Show that the potential function

\[ \phi = \left( \frac{1}{3} \right) x^3 - x^2 - xy^2 + y^2 \]

describes the irrotational flow. Determine the stream function and the velocity vector at \((1, 2)\).

12. (a) Draw the flow net for a uniform two-dimensional flow with a constant velocity \( u = c \).

(b) What arrangement of sources and sinks will give rise to the function

\[ \omega = \log \left( z - \frac{a^2}{z} \right) \]?
(b) \( \frac{m \dot{h}^2}{p^2} \frac{dp}{dn} = -F \)

(c) अध्ययनार्थ चिह्नन करोः.

11. (a) \( \text{Dimensional} \) उदाहरण \( \phi = \frac{1}{3} x^3 - x^2 - xy^2 + y^2 \) गणितीय \( \text{अध्ययनार्थ चिह्नन} \) करोः।

(b) \( \dot{\phi} = \frac{x^3}{3} - x^2 - xy^2 + y^2 \) गणितीय \( \text{अध्ययनार्थ चिह्नन} \) करोः।

12. (a) \( u = c \) \( \text{अवकल} \) \( \text{रचना} \) \( \text{करोः} \) \( \text{समीकरण} \) \( \text{समाधाने} \) \( \text{करोः} \)

(b) \( \dot{w} = \log \left( z - \frac{a^2}{z} \right) \) \( \text{दर्शने} \) \( \text{करोः} \) \( \text{स्रोते} \) \( \text{सौंदर्ये} \) \( \text{करोः} \)

| Turn over |
13. (a) Fit a cubic spline function for the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

(b) Evaluate \( \int_{0}^{6} \frac{dx}{1 + x^2} \) by using (i) Trapezoidal rule, (ii) Simpson's \( \frac{1}{3} \) rule, (iii) Simpson's \( \frac{3}{8} \) rule, (iv) Weddle's rule and compare the result with its actual value. (Divide the interval \( (0, 6) \) into six parts each of width \( h = 1 \)).

(c) The angular displacement at different times is given by the following table. Find the angular velocity and angular acceleration at \( t = 2.5 \) sec.

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>0</td>
<td>1.01</td>
<td>2.16</td>
<td>3.81</td>
<td>6.56</td>
<td>11.25</td>
</tr>
</tbody>
</table>

14. (a) Consider the function \( f(x) = e^x \) at the points \( x = 0, 0.5, 1 \). Using numerical differentiation, find \( f''(0) \). Estimate the error bound and compare it with the actual error.

(b) A slider in a machine moves along a fixed straight rod. Its distance \( x \) cm along the rod is given below for various values of the time \( t \) seconds. Find the velocity of the slider and its acceleration when \( t = 0.3 \) sec.

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>30.13</td>
<td>31.62</td>
<td>32.87</td>
<td>33.64</td>
<td>33.95</td>
<td>33.81</td>
<td>33.24</td>
</tr>
</tbody>
</table>

(c) Discuss the convergence criteria for Newton-Raphson's method.
13. (a) दिनेक क्रमांकातील लोके शॉट $x_i$ द्वारा (Spline) दर्शेलेले असलेले:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

(b) (i) अंतरबांधकर (ii) अंतराळी $\frac{1}{3}$ (iii) अंतराळी $\frac{2}{3}$ (iv) अंतराळी 1 होणारे, निम्नलिखित 6

\[ \int_{0}^{6} \frac{dx}{1 + x^2} \]

(0, 6) अंतराळीपक्षे त्याचे मानणे मजलित होते. तुम्ही आपणे अनुसार $h = 1$ निष्ठाने।

(c) धार्मिक अनुसूचित क्रमांकातील अवस्थानाची (Displacement) दर्शन करून केलेले होते. दरासी करून तत्वांना प्रत्येक अवस्थानाची, $t = 2.5$ लक्षणांक, दर्शनकरते.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0</td>
<td>1.01</td>
<td>2.16</td>
<td>3.81</td>
<td>6.56</td>
<td>11.25</td>
</tr>
</tbody>
</table>

14. (a) $x = 0, 0.5, 1$ विशेषत्वात, $f(x) = e^x$ अरूंदीत प्रक्रिया असलेलेली, $x = 0, 0.5, 1$ (निर्देशित)

याची त्रिकोणमितीय $f'(0)$ दर प्रतिक्रिया. तुमच्या बदलाच्या अभावाने मान असणारे अरूंदीत अवस्थावरून तुमच्या बदलाच्या अभावाने मान असणारे.

(b) धार्मिक अनुसूचित $x$ द्वारे $y$ अरूंदीत अवस्थानाचे वृत्तांकाचे विश्लेषण. तुमच्या बदलाच्या $x$ अवस्थानाने. केलेले, $x = t$ अरूंदीत अवस्थानाचे वृत्तांकाचे विश्लेषण. $t = 0.3$ लक्षणांक

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>30.13</td>
<td>31.62</td>
<td>32.87</td>
<td>33.64</td>
<td>33.95</td>
<td>33.81</td>
<td>33.24</td>
</tr>
</tbody>
</table>

(c) निर्देशित-अनुसूचित (Criteria) नावाने माने.
15. (a) Calculate the mean deviation from median for the following:

<table>
<thead>
<tr>
<th>Rs. per day</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of</td>
<td>2</td>
<td>9</td>
<td>11</td>
<td>14</td>
<td>20</td>
<td>25</td>
<td>24</td>
<td>23</td>
<td>20</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>families</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Let $X$ and $Y$ be two random variables taking three values $-1, 0, 1$ and having the joint probability distribution given below:

<table>
<thead>
<tr>
<th>$Y \downarrow$</th>
<th>$X \rightarrow$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$0$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$0$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$1$</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(i) Given that $Y = 0$, what is the conditional probability distribution of $X$?

(ii) Show that $X$ and $Y$ have different expectations.

(iii) Prove that $X$ and $Y$ are uncorrelated.

(iv) Find $\text{Var } X$ & $\text{Var } Y$.

16. (a) Find an optimal solution to the following L.P.P. by computing all basic solutions and then finding one that maximizes the objective function:

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Max $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$.

(b) In a factory, there are six jobs to perform and each should go through two machines $A$ and $B$ in the order $A, B$. The processing timings in (hours) for the jobs are given below. Determine the sequence for performing the jobs that would minimize the total elapsed time $T$. What is the value of $T$?

<table>
<thead>
<tr>
<th>Job :</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
<th>$J_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine $A$</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Machine $B$</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>
15. (a) Fill in the following table:

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>11</td>
<td>14</td>
<td>20</td>
<td>25</td>
<td>24</td>
<td>23</td>
<td>20</td>
<td>16</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(b) Given the data set $-1, 0, 1$, calculate the values of $X$ and $Y$ and determine if $X$ is a function of $Y$. Also, determine the expectations of $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.4</td>
<td>1</td>
</tr>
</tbody>
</table>

(i) Is $Y = 0$ a solution, $X$ is a function (Conditional) of $Y$? Explain.
(ii) $X$ and $Y$ are independent (Conditions) of $Y$.
(iii) Is $X$ an expectation of $Y$?
(iv) $Var X = Var Y$.

16. (a) A teacher is evaluating the effectiveness of a new teaching method $T$. To determine its effectiveness, he conducted an L.P.P. problem and the objective function is:

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Maximize, $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$.

(b) The teacher's teaching method $T$ is evaluated against a traditional teaching method $A$ and $B$. The criteria for evaluation are $J_1, J_2, J_3, J_4, J_5, J_6$.

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
<th>$J_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>$A$</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>