INSTRUCTIONS

Each question is printed both in English and in Kannada.

Answers must be written in the medium specified (English or Kannada) in the Admission Ticket issued to you, which must be stated clearly on the cover of the answer-book in the space provided for this purpose. No credit will be given for the answers written in a medium other than that specified in the Admission Ticket.

The Question Paper is divided into three Sections A, B and C.

Candidates should attempt any five questions choosing at least one but not more than two from each Section.

Assume suitable data, if considered necessary, and indicate the same clearly.

All questions carry equal marks.
SECTION - A

(Probability)

1. (a) Show that the conditional probability function \( P(\cdot | E) \) satisfies the axioms of a probability space. Demonstrate through an example that pair-wise independence need not imply mutual independence.

(b) Two dice, one green and the other red are thrown. Let \( A \) be the event that the sum of points of the faces shown is odd and \( B \) the event of at least one ace (number 1). Describe the sample space and event space \( A \cap B \) and \( A \cap \bar{B} \) and find the probabilities of the events \( P\left( \bar{A} \cup B \right) \), \( P\left( \bar{A} \cap (A \cup B) \right) \), \( P\left( B \mid A \right) \).

(c) Let \( r \) indistinguishable balls be placed at random in \( n \) components of a box. Let \( A_k \) be the event that a specified compartment has exactly \( k \) balls. Find \( P\left( A_k \right) \).

2. (a) The joint density function of the random variable \( X \) and \( Y \) is given by

\[
f(X, Y) = \begin{cases} 
8xy & 0 \leq x \leq 1; 0 < y \leq x \\
0 & \text{otherwise}
\end{cases}
\]

Find (i) the marginal density of \( X \) (ii) the marginal density of \( Y \) (iii) the conditional density of \( X \) and (iv) the conditional density of \( Y \).

(b) For a given \( \lambda \) \((> 0)\), the random variable \( X \) has the Poisson distribution with parameter \( \lambda \). But \( \lambda \) itself is a random variable having pdf

\[
g(\lambda) = \frac{\lambda^r}{r!} \exp\left(-\lambda \right) \cdot \lambda^{r-1}, \lambda \geq 0.
\]

What is the unconditional distribution of \( X \)? Obtain the mean and variance of this distribution. Are the two still equal?

(c) Let \( f(x_1, x_2) = 21 x_1^2 x_2^2 ; 0 < x_1 < x_2 < 1 \) and zero elsewhere be the joint p.d.f. of \( X_1 \) and \( X_2 \). Find the conditional mean and variance of \( X_1 \) given \( X_2 = x_2 \); \( 0 < x_2 < 1 \).
1. (a) Let \( P(\emptyset | E) \) denote the conditional probability of an event \( \emptyset \) given event \( E \). If \( \emptyset \) is a subset of \( E \), then \( P(\emptyset | E) = P(\emptyset) \). If \( \emptyset \) and \( E \) are disjoint, then \( P(\emptyset | E) = 0 \).

(b) Let \( A \) and \( B \) be events. If \( A \subseteq B \), then \( P(A) \leq P(B) \). If \( A \) and \( B \) are disjoint, then \( P(A \cup B) = P(A) + P(B) \).

(c) Let \( A \) and \( B \) be events. If \( A \cap B = \emptyset \), then \( P(A \cup B) = P(A) + P(B) \).

2. (a) Let \( X \) and \( Y \) be random variables defined on the same probability space as follows:

\[
f(X, Y) = \begin{cases} 8xy & 0 \leq x \leq 1, 0 < y \leq x \\ 0 & \text{otherwise} \end{cases}
\]

(i) Let \( \lambda > 0 \) be a parameter. (ii) Show that \( f(X, Y) \) is a probability density function. (iii) Find the marginal density function of \( X \). (iv) Find the marginal density function of \( Y \).

(b) Let \( X \) and \( Y \) be random variables. Let \( \lambda > 0 \) be a parameter. Let \( g(\lambda) = \frac{\alpha^{r}}{\Gamma(r)} \exp(-\alpha \lambda), \lambda > 0 \). Find the joint distribution of \( X \) and \( Y \).

(c) Let \( f(x_1, x_2) = 21x_1^2x_2^2 \) for \( 0 < x_1 < x_2 < 1 \). Let \( X_1, X_2 \) be random variables with the given probability density function (p.d.f.). Let \( X_1 \) and \( X_2 \) be independent random variables. Let \( X_2 = x_2 \) for \( 0 < x_2 < 1 \).
3. (a) State the continuity theorem for characteristic functions. Let $X_n$ be a random variable with p.m.f.

$$P\left( X_n = 1 \right) = \frac{1}{n} ; \quad P\left( X_n = 0 \right) = 1 - \frac{1}{n}$$

Investigate whether the sequence of distribution functions of $X$ converges weakly to the distribution function of a random variable. If yes, find the distribution of the limiting random variable.

(b) Establish basic inequality and demonstrate its use in obtaining Markov inequality.

(c) If $X$ and $Y$ are independent Gamma variates with parameters $\mu$ and $\gamma$ respectively, show that

$$U = X + Y, \quad Z = \frac{X}{Y}$$

are independent and that $U$ is gamma ($\mu + \gamma$) variate and $Z$ is a $\beta_2 (\mu, \gamma)$ variate.

4. (a) Explain Law of Large Numbers. State and prove Khintchine's theorem. Let $X_i$ assume two values 1 and -1 with equal probabilities. Show that the law of large numbers cannot be applied to these variables.

(b) State and prove the De Moivre-Laplace theorem.

(c) If $\{ X_k \}$ be a sequence of random variables such that

$$\frac{1}{n^2} \text{Var} \left( \sum_{1}^{n} X_k \right) \rightarrow 0 \text{ as } n \rightarrow \infty$$

then show that $\bar{X}_n - \bar{\mu}_n \rightarrow 0$ where $E X_k = \mu_k$. 
3. (a) The random variable $X_n$ is the number of successes in $n$ trials. $X_n$, p.m.f. depends on the probability of success.

$$P\left( X_n = 1 \right) = \frac{1}{n}; \quad P\left( X_n = 0 \right) = 1 - \frac{1}{n}$$

X is binomial distributed, and the probability of success in each trial is constant, and success is independent of the number of previous successes. Therefore, the distribution of $X_n$ is well-specified.

(b) The random variable $X_n$ is the sum of $n$ independent random variables $X_1, \ldots, X_n$, each with the same distribution.

(c) Let $Y = X + Y$, $Z = \frac{X}{Y}$, and $W = X_2$. Then $W$ is a gamma distribution.

4. (a) The random variable $X_n$ is the sum of $n$ independent random variables $X_1, \ldots, X_n$. Each $X_i$ is a gamma distributed random variable. $X_n$ is also gamma distributed.

(b) As $n$ increases, the distribution of $X_n$ approaches a normal distribution.

(c) The sequence of random variables $\left\{ X_k \right\}$ satisfies $\frac{1}{n^2} \text{Var} \left( \sum_{i=1}^{n} X_k \right) \to 0$ as $n \to \infty$. Therefore, $E(\bar{X}_n) = \mu_k$ and $\bar{X}_n - \mu_n \to 0$ as $n \to \infty$. Therefore, $EK_k = \mu_k$ and $\bar{X}_n - \mu_n \to 0$ as $n \to \infty$. [Turn over]
SECTION - B

(Statistical Inference)

5. (a) Explain the concept of estimation in statistical inference and record your understanding of consistency and efficiency.

(b) Let \( X_1, X_2, \ldots, X_n \) be a sample from

\[
f(x) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq x \leq \theta_2 \\ 0, & \text{otherwise} \end{cases}
\]

Obtain the estimates of \( \theta_1, \theta_2 \) by the method of moments and by the method of maximum likelihood.

(c) State and explain Lehmann-Scheffe theorem with an example.

6. (a) Explain the concept of (i) Type II error and (ii) Power curve. Let \( X \) has a binomial law with \( n = 10, p \) where \( pt \left( \frac{1}{2}, \frac{1}{4} \right) \). If the observed value of \( X_1 \), a random sample of size one, is less than or equal to 3, we reject \( H_0 : p = \frac{1}{2} \) and accept \( H_1 : p = \frac{1}{4} \). Find the power function of the test.

(b) Develop the test procedure to test the hypothesis \( H_0 : \mu = \mu_0 \) against \( H_1 : \mu = \mu_1 \) in the distribution

\[
f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \text{ where } \sigma \text{ is known.}
\]

(c) Let \( \{X_1, X_2, \ldots, X_n\} \) be a random sample from a distribution with p.d.f.

\[
f(x; \theta) = \frac{1}{\theta}; \quad 0 < x < \theta. \text{ Find the confidence interval for } \theta \text{ based on maximum likelihood estimator of } \theta.
\]
5. (a) Let \( X \) be a continuous random variable with density function \( f(x) \). Find \( f(x) \) for \( x \in (\theta_1, \theta_2) \).

(b) \( f(x) = \begin{cases} 
\frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq x \leq \theta_2 \\
0, & \text{otherwise}
\end{cases} \)

(c) Explain the meaning of \( f(x) \) in the context of the problem.

6. (a) Let \( X \) be a discrete random variable. Find the probability mass function (pmf) of \( X \) for \( n = 10 \) trials.

(b) Find the expected value of \( X \), denoted as \( E(X) \).

(c) Find the variance of \( X \), denoted as \( Var(X) \).
7. (a) Let $X_1, ..., X_n$ and $Y_1, ..., Y_m$ be random samples from the independent distributions $N(\theta_1, \theta_3)$ and $N(\theta_2, \theta_4)$ respectively. Show that the likelihood ratio that $H_0: \theta_3 = \theta_4, \theta_1, \theta_2$ unspecified against $H_1: \theta_3 \neq \theta_4, \theta_1, \theta_2$ unspecified can be based on the random variable:

$$F = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2 / (n - 1)}{\sum_{i=1}^{m} (Y_i - \bar{Y})^2 / (m - 1)}$$

(b) State Neyman-Pearson lemma. Mention its restrictions. A sample of size 1 is taken from density

$$f(x; \theta) = \begin{cases} \frac{2(x - \theta)}{\theta^2} & ; \theta < x < \theta \\ 0 & \text{elsewhere} \end{cases}$$

Find the MPT of $H_0: \theta = \theta_0$ versus $H_1: \theta = \theta_1 \ (\theta_0 > \theta_1)$ at level $\alpha$.

(c) Let $k$ independent random samples be drawn from $k N(\theta_i, \sigma_i^2)$ distributions, $i = 1, 2, ..., k$. Derive the LR test for $H_0: \theta_1 = \theta_2 = ... = \theta_k$ assuming $\sigma_1 = \sigma_2 = ... = \sigma_k$. Show that this test can be reduced to the $F$-test.

8. (a) Define the OC function and ASN function of sequential analysis. Derive then appropriate expression for the sequential probability ratio test of a simple hypothesis against a simple alternative.

(b) Describe clearly (i) Wilcoxon-Mann-Whitney test and (ii) Kolmogorov-Smirnov test for the two sample problem. Consider both one-sided and two-sided alternatives.

(c) Indicate the details of comparison of the A.S.N. of S.P.R.T. with that of fixed sample of size $n$ of Neyman-Pearson test of hypothesis.
7. (a) $N(\theta_1, \theta_4)$ and $N(\theta_2, \theta_4)$ distributions are independent of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$. Let

$$F = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)}{\sum_{i=1}^{m} (Y_i - \bar{Y})^2 / (m-1)}$$

be the F-test statistic, where $H_1: \theta_3 \neq \theta_4$, $\theta_1 \neq \theta_2$ and $H_0: \theta_3 = \theta_4$, $\theta_1 = \theta_2$. If $\alpha$ is the significance level, the $f(x; \theta) = \begin{cases} \frac{2(\theta - X)^2}{\theta^2} & ; 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$ distribution is used as the null distribution of $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ ($\theta_0 > \theta_1$) under MPT.

(b) Consider the following Lemma (Lemma): Let $X_1, \ldots, X_n$ be a random sample from $f(x; \theta)$, where $f(x; \theta) = \begin{cases} \frac{2(\theta - x)}{\theta^2} & ; 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$ for $0 < \theta < \infty$. The probability density function of $\theta$ is $\Gamma(\alpha, \theta)$, where $\alpha > 0$ is the shape parameter and $\theta > 0$ is the scale parameter. The likelihood ratio test statistic is given by $L = \frac{L_1}{L_0}$, where $L_1$ is the maximum likelihood estimate of $\theta$ under the alternative hypothesis and $L_0$ is the maximum likelihood estimate of $\theta$ under the null hypothesis.

(c) $k N \left(\theta_i, \sigma_i^2\right)$ for $i = 1, 2, \ldots, k$ are independent and identically distributed. Let $\sigma_1 = \sigma_2 = \ldots = \sigma_k$, and $H_0: \theta_1 = \theta_2 = \ldots = \theta_k$. The likelihood ratio test statistic is given by $L = \frac{L_1}{L_0}$, where $L_1$ is the maximum likelihood estimate of $\theta$ under the alternative hypothesis and $L_0$ is the maximum likelihood estimate of $\theta$ under the null hypothesis. The test statistic is distributed as $(\chi^2_k - \chi^2_k)$.

8. (a) Consider the following OC curve for the ASN function, where $a > 0$ is a constant. The OC curve is given by $y = \frac{a}{x}$ for $x > 0$. When the ASN function is used, the OC curve is given by $y = \frac{a}{x}$ for $x > 0$. The ASN function is defined as $ASN = \int_0^\infty \frac{a}{x} \, dx$.

(b) Consider the following properties: (i) The OC curve for the ASN function is decreasing. (ii) The ASN function is concave up for $x > 0$. The ASN function is defined as $ASN = \int_0^\infty \frac{a}{x} \, dx$ for $x > 0$.

(c) Under the hypothesis that the distribution is $N$ or $\Gamma$, the S.P.R.T. is $\text{S.P.R.T.}$,
SECTION – C

(Linear Inference and Multivariate Analysis)

9. (a) Prove the necessary and sufficient condition for a parametric function to be estimable and that for a linear function of the variable belongs to error. Show that BLUEEs are uncorrelated with linear functions belonging to error.

(b) Let \( y_i = \beta_1 x_i + \beta_2 x_i + e_i \); \( i = 1, 2, \ldots, n \) be a simple linear model where \( \beta_1 \) and \( \beta_2 \) are unknown scalar constants, \( x_i \) are known scalar constants. Develop a test procedure to obtain the confidence interval for

\[
\bar{y}_0 = \frac{1}{k} \sum_{j=1}^{k} y_{0j}.
\]

(c) Use the technique of analysis of variance for testing that in a multiple linear regression model there is no dependence of the dependent variable on the regressor variable.

10. (a) Show that the general linear regression model covers both multiple linear regression and polynomial regression. Show how this fits into Gauss-Markov linear model.

(b) If the \( n_y \) in the model \( y_{ijk} = \mu + \tilde{z}_i + \tilde{\beta}_j + e_{ijk} \) are such that

\[
\tilde{z}_i - \tilde{z}_j: \tilde{\beta}_i - \tilde{\beta}_j, \text{are estimable for all } i \neq j \text{ and all } i' \neq j', \text{then}
\]

(i) there are exactly \( b + t - 1 \) linearly independent estimable functions

(ii) \( \sum c_i \tilde{z}_i \) and \( \sum d_j \tilde{\beta}_j \) are estimable if \( \sum c_i = \sum d_i = 0 \).

(c) Provide the details of multiple correlation coefficient and its estimation. Obtain the distribution of the sample multiple correlation coefficient when population multiple correlation coefficient is zero.
( Karnataka: ಕರ್ನಾಟಕ: )

10. (a) ಎನ್ನುವಾಗಿ (ಸ್ವತ್ತ) ಜೋಡಿಸಿಕೆಯು ಇದೆ ಇದೆ ಅದರ ಸಂಖ್ಯೆಯನ್ನು ಕಲನ್ನುಂಟುಕೊಂಡು ಪಡೆಬೇಕು. ಇದರ ಮೇಲೆ ಜೋಡಿಸಿಕೆಯು ತನ್ನ ಪಾಡುವಿಕೆಯನ್ನು ಉಪಯೋಗಿಸಿ ಇತರ ಮೇಲೆ ಜೋಡಿಸಿಕೆಯ ಒಳಗೆ ಅನುಕೂಲವಾಗಿರುವ ಪರಿಣಾಮವನ್ನು ಪಡೆಯುತ್ತದೆ. ಎಂಕಿ ಆಸಕ್ತಿಯಾಗಿ ಸಹಾಯ ಮಾಡಲಾಗುತ್ತದೆ.

(b) \[ y_i = \beta_1 + \beta_2 x_i + e_i; \quad i = 1, 2, \ldots, n \]

(c) ಲೋಕ ಮಾರ್ಗವಾಹಿಯಲ್ಲಿ ಪ್ರತ್ಯೇಕ ಗಣಾಪತಿಯನ್ನು ಸುಲಭವಾಗಿ ಮಾಡಿದಂತೆ ಈ ಮೇಲೆ ಪರಿಣಾಮವನ್ನು ಉಪಯೋಗಿಸಿ ಯೊಂದಿಗೆ ಮಾರ್ಗವಾಹಿ ಗೊತ್ತಿರುವ ಇತರ ಮೇಲೆ ಜೋಡಿಸಿಕೆಗಳನ್ನು ಸಹಾಯಿಸುತ್ತದೆ.

11. (a) ಎಂದರೆ, ಶಾಲೆಗೆ ಮಾರ್ಗವಾಹಿಯಲ್ಲಿ ಪ್ರತ್ಯೇಕ ಗಣಾಪತಿಯನ್ನು ಸುಲಭವಾಗಿ ಮಾಡಿದಂತೆ ಈ ಮೇಲೆ ಪರಿಣಾಮವನ್ನು ಉಪಯೋಗಿಸಿ ಯೊಂದಿಗೆ ಮಾರ್ಗವಾಹಿ ಗೊತ್ತಿರುವ ಇತರ ಮೇಲೆ ಜೋಡಿಸಿಕೆಗಳನ್ನು ಸಹಾಯಿಸುತ್ತದೆ.

(b) \[ y_{i,j} = \mu + \bar{z}_i + \beta_j + e_{i,j} \]

(i) \[ b + i - 1 \]

(ii) \[ \sum_{i=1}^{n} y_{i,j} = 0 \]

(c)  ಸಾಮಾನ್ಯತೆಯಾದ ಬಿದ್ದೆಲೆಯನ್ನು ಹುಲ್ಲಾದಾಗ ಸಹಾಯ ಮಾಡುವುದರಿಂದ, ಈ ತುದಿಯು ಸಾಮಾನ್ಯತೆಯಾದ ಬಿದ್ದೆಲೆಯನ್ನು ಹುಲ್ಲಾದಾಗ ಸಹಾಯ ಮಾಡುವುದರಿಂದ, ಈ ತುದಿಯು.
11. (a) Show that the matrix in the exponent of a \( p \)-variate normal distribution is the inverse of the variance-covariance matrix of the vector \( Y \).

(b) If \( X_1, \ X_2, \ldots, \ X_n \) are sample means for random sample of size \( n \) from \( N_p(\mu, \Sigma) \), show that \( n \left( \bar{X} - \mu \right) \Sigma^{-1} \left( \bar{X} - \mu \right)' \) has the \( \chi^2 \)-distribution with \( p \) degrees of freedom.

(c) Show that under usual normal population regression model the joint distribution of the partial regression coefficient is multivariate normal.

12. (a) Provide the description of Fisher's discriminant function and develop the test procedure for additional discrimination and assigned discriminant functions.

(b) Define Hotelling's \( T^2 \)-statistic. Derive the likelihood ratio criterion for testing the hypothesis that the mean vectors of two \( p \)-variate normal distributions with same dispersion matrix, are equal. Show that the above test can be given by \( T^2 \)-statistic.

(c) Provide a detailed note on the basic concepts of principal component analysis.
11. (a) $p$-ನೇ ಸಂಖ್ಯೆ, ಮಾದರಿಯಾದ $X_1, X_2, \ldots, X_n$ ಎಂಬುದು ಮಾರಾಟಕ್ಕೆ ಸರಳ ಸಮಸ್ಯೆಯಲ್ಲಿ ಮೇಲೆ ನೂತನ
ಎಂಬುದು.

(b) $N_p (\mu, \Sigma)$ ಮೇಲೆ ಸೂತ್ರ $n$ ಸರಳರೂಪದ ಎಳೆ ಹಾಗೂ $X_1, X_2, \ldots, X_n$ ಮೇಲೆ ಏರಿಕೆಯಾದ $\Sigma^{-1} (X - \mu)$ ಎಂಬ ಪ್ರತಿ]<=

(c) ಸೂತ್ರವು ಎಲ್ಲ ರೀತಿಯಾದ ಸರಳರೂಪದ ಎಳೆಗಳಿಗೆ ನಿರ್ದೇಶಿಸುತ್ತದೆ $

12. (a) $q$-ನೇ ಸಂಖ್ಯೆ, ಸರಳರೂಪದ ಲೋಹಾತ್ಮಕ ಲಭ್ಯವಾದ ಮೇಲೆ ಮಾರಾಟಕ್ಕೆ ಸರಳ ಸಮಸ್ಯೆಯಲ್ಲಿ ಮೇಲೆ ನೂತನ $\Sigma^{-1} (X - \mu)$

(b) $T^2$ ಎಂಬ ಸೂತ್ರವು $N_p (\mu, \Sigma)$ ಮೇಲೆ ಈ ಜೀಡಿಯ ಸರಳರೂಪದ $\Sigma^{-1} (X - \mu)$

(c) $\Sigma^{-1} (X - \mu)$ ಎಂಬ ಪ್ರತಿ $p$-ದ ಸೂತ್ರವು $N_p (\mu, \Sigma)$ ಮೇಲೆ ಎಳೆಗಳಿಗೆ ಈ ಜೀಡಿ ಎಂಬ ಸರಳರೂಪದ $\Sigma^{-1} (X - \mu)$.
2002

STATISTICS

Paper II

[Maximum Marks: 300]

INSTRUCTIONS

Each question is printed both in English and in Kannada.

Answers must be written in the medium specified (English or Kannada) in the Admission Ticket issued to you, which must be stated clearly on the cover of the answer-book in the space provided for this purpose. No credit will be given for the answers written in a medium other than that specified in the Admission Ticket.

Candidates should select any three Sections and attempt any five questions from the selected Sections, choosing at least one but not more than two questions from each of the selected Sections.

Assume suitable data, if considered necessary, and indicate the same clearly.

All questions carry equal marks.
SECTION – A
(Sampling Theory and Design of Experiments)

1. (a) Explain the terms (i) proportional allocation (ii) Neyman allocation and (iii) optimum allocation as used in stratified sampling. Make a critical comparison of these methods.

(b) State the Horvitz-Thompson estimator of a population total in PPS sampling without replacement. Show that it is unbiased.

(c) Where there is a linear trend in the population for the study variable, show that the efficiency of the systematic sample is \( n \) (sample size) times that of simple random sampling without replacement.

2. (a) Define Ratio estimator. Derive the approximate variance of this estimator in terms of coefficient of variation and relative covariance.

(b) Explain the use of three stage sampling in sample surveys. Obtain an unbiased estimator of the population mean and its variance in the case if simple random sampling is used in all three stages.

(c) Provide a critical note with suitable examples on 'non-sampling errors'.

3. (a) State the basic principles of design of experiments and their role in such experiments.

(b) Define an RBD. Give the linear model and outline the method of analysis of an RBD.

(c) In an LSD the observation in the \( i^{th} \) row, \( j^{th} \) column, receiving the \( k^{th} \) treatment is missing. Obtain a valid estimate of this observation. What will you do if more than one observation is missing? How will the subsequent analysis change?

4. (a) What are factorial experiments? Illustrate the concept of total confounding in such experiments.

(b) Define BIBD. Derive the following parametric relations:

(i) \( vr = bk \)  
(ii) \( \lambda (v - 1) = r(k - 1) \)  
(iii) \( b \geq r \) (Fisher's Inequality).

(c) Assuming a suitable model, develop the analysis to test various effects in the case of split-plot design with RBD layout for main plot treatment.
( ಹೆಸರು ರೀತಿಯಲ್ಲಿ ಕೆಲಸಿದ್ದ ಭಾಷೆಯನ್ನು ಎಣ್ಣೆ )

2. (a) ದಿಂದುವಿನ ವಿಧಗಳು ಹಾಗು ಸ್ಥಳಗಳನ್ನು ಪ್ರಕಟಿಸಿ.
(b) ಸ್ತೂಪುಗಳಲ್ಲಿ ಲೆಬೆಲುಗಳನ್ನು ಹಾಗು ತಂತ್ರಗಳನ್ನು ಸೇರಿಸಿ.
(c) ಇದ್ದುವಿನ ವಿಧಗಳನ್ನು ಹಾಗು ಸ್ಥಳಗಳನ್ನು ಪ್ರಕಟಿಸಿ.

3. (a) ಶಾಸ್ತ್ರ ಸಂಬಂಧಿಯಿರುವ ವ್ಯಕ್ತಿಗಳು ಹಾಗು ನಿರ್ದೇಶನಗಳು ಹಾಗು ಸೇರಿಸಿ.
(b) ಪ್ರಾಯೋಜನದ ವಿಧಗಳನ್ನು ಹಾಗು ಸ್ಥಳಗಳನ್ನು ಪ್ರಕಟಿಸಿ.
(c) ಇದ್ದುವಿನ ವಿಧಗಳನ್ನು ಹಾಗು ಸ್ಥಳಗಳನ್ನು ಪ್ರಕಟಿಸಿ.

4. (a) ಬಲವಿನ ಈಸವಿಯಿಂದ ಹಾಗು ಸರಿಯಾದ ಹೊಂದಿಕೆಗಳು ಹಾಗು ಸೇರಿಸಿ.
(b) BIBD ಹಾಗು ಸೇರಿಸಿ.
(c) ತನ್ನ ಪ್ರತ್ಯೇಕಿಸಿರುವ RBD ಕ್ರಮದ ಮೇಲೆ ಅಧಿಕಾರ ಜಾತಿಗಳು ಹಾಗು Split-plot design ಹಾಗು ಹಾಗು ಹಾಗು ಸೇರಿಸಿ.
SECTION - B

(Engineering Statistics)

5. (a) Explain the significance of the following concepts in the construction of control charts:
   (i) Assignable causes of quality variation
   (ii) Rational subgroups.

(b) Explain the following for $\bar{X}$ and $R$ charts:
   (i) Objectives
   (ii) Statistical basis and construction
   (iii) Inference from charts.

(c) Quality control charts are to be prepared with a sensitivity that a mean shift of 0.6 $\sigma$ will cause $\frac{2}{5}$ of samples to fall outside the mean chart warning limit of 1.96 sigma control limit. What sample size is advisable?

6. (a) What is acceptance sampling? Mention the situations where it is useful. What are the advantages and disadvantages?

(b) State the meaning and utility of the following concepts in acceptance sampling:
   (i) OC
   (ii) AQL
   (iii) AOQL.

(c) Write down the operating procedures and expressions for different parameters of sequential sampling plan.

7. (a) Define a parallel system. Obtain its reliability function and mean life of the system if all the units are equally reliable and reliability law is exponential.

(b) Identify the following distribution:

$$f(x | \sigma : p) = \frac{p}{\sigma} x^{p-1} \exp \left\{ -\frac{x^p}{\sigma} \right\}, \quad x > 0, \ p > 0, \ \sigma > 0$$

as a failure time distribution and detail the procedure of estimating its parameters.

(c) Let $X_{(1)} < X_{(2)} < \ldots < X_{(r)}; \ (r < n)$ denote the ordered life-times observed where $n$ units are on test, each following an exponential distribution with mean $\lambda$. Obtain the UMVUE of reliability at a given time point.
(অনেক প্রশ্নের উত্তর)

1. বিশ্বাস সৃষ্টির জন্য একটি স্মৃতি সৃষ্টির জন্য কিভাবে প্রয়োগ করা হয়?

2. কীভাবে মানমন্ত্র সম্ভব হয়?

3. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

4. $X_1, X_2, \ldots, X_n$ আল্পসাধারণ $X$ এর উপর একটি সম্পূর্ণ দৃষ্টিকোণ হলো:

5. কীভাবে মানমন্ত্র সম্ভব হয়?

6. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

7. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

8. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

9. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

10. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

11. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

12. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

13. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

14. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

15. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

$f(x | \sigma; \mu) = \frac{1}{\sigma} x^{p-1} \exp \left\{ -\frac{x^p}{\sigma} \right\}, \ x > 0, \ p > 0, \ \sigma > 0$

16. কীভাবে মানমন্ত্র ফলাফলের সংখ্যা বৃদ্ধি করা যায়?

[Turn over]
8. (a) Describe the system reliability in terms of (i) components connected in series, (ii) components connected in parallel and (iii) mixed system.

(b) Given the hazard function \( h(t) = \lambda t, \lambda > 0 \), compute the reliability function and the mean time to failure.

(c) Explain the concept of parallel redundancy and state the application of binomial distribution to parallel redundancy cases.
(a) (i)  ಸ್ವತಃ ಸ್ವತಃಭದ್ರ ಜೀವನದ ಚಿತ್ತುಬಂದಿ
(ii) ಕನಂಜುವಿನ ಜೀವನ ನಿವಾಸ ವಿಧಾನ
(iii) ಕನಂಜುಗಳು; ಕನಂಜುಗಳು ಪ್ರತಿಯೊಂದು ಸುಧಾ ಭಕ್ತಿ
(b) ಕೋನುಗುಣದ ರೊಳಗುವಳಿ ಫಲ h ( t ) = \lambda t, \lambda > 0 ಎಂಬುದುಗಳು ಕೋಂಜುರೋಳಗುವಳಿ ಫಲವಾಗುತ್ತದೆ. ಇದು ಬೆಳಕಿ, ಕೋಂಜು, ರೊಳಗಳು.
(c) ಕೋಂಜುರೋಳಗುವಳಿ ಕೋಂಜುರೋಳಗುವಳಿ ಇಂದು ಮೇಲೆ, ಕೋಂಜುರೋಳಗುವಳಿ ಕೋಂಜು ರೊಳಗುವಳಿ ಇಂದು ಮೇಲೆ.
SECTION - C
(Operations Research)

9. (a) Define:

(i) Markov chain
(ii) Persistent state
(iii) Ergodic state.

Show that in a finite Markov chain there exist no null states and that it is impossible that all states are transient.

(b) What do you understand by higher transition probabilities? Obtain the Chapman-Kolmogorov equation.

(c) Derive the steady state results (probabilities, mean numbers and mean waiting times) for an $M|M|S$ queue.

10. (a) What is Operations Research and what are the essential characteristics of Operations Research?

(b) Define (i) basic feasible solution (ii) slack and surplus variables and (iii) artificial variables in the context of a Linear Programming Problem (LPP).

(c) State the general linear programming problem. Show that the collection of all feasible solutions of a linear programming constitute a convex set.

11. (a) State the $(s,S)$ policy in an inventory problem. Derive this policy when the demand has a negative exponential distribution.

(b) Explain the replacement problem and describe the problem of replacement of items whose maintenance cost increases with time and value of the money remains constant.

(c) Describe the transportation problem. Show that the necessary and sufficient condition for the existence of a feasible solution to the transportation problem is the total capacity $\left(\sum_{i=1}^{m} a_i\right)$ must be equal to the requirement $\left(\sum_{j=1}^{n} b_j\right)$. 
(iii) Persistent state

(iii) Ergodic state

(Operations Research)

(iii) LPP
12.  (a) Describe and justify the algorithm used to find the optimal solution for a transportation problem by the modified stepping stone method.

(b) Write a Function sub-program to evaluate the factorial of an integer $k$ and use it to compute and print out the values of the binomial co-efficients

$$\binom{N}{R} = \frac{N!}{R! \cdot (N-R)!}$$

for 50 sets of values of $N$ and $R$. Assume $N, R \geq 0$.

(c) Write a statement function defining $\text{SUM}(A, B, C) = \sqrt{A + B + C}$ and use this function to compute $\alpha = \sqrt{a^2 + b^2 + c^2}$; $\beta = \sqrt{7x + 3y - 8}$. 
12. (a) Let $a$ be a positive integer. Define $k = a + b + c$. Then find the value of $a$ using the given algorithm.

(b) Let $k$ be a positive integer. Define $N$ as $2k$. Then find the value of $R$ using the given algorithm.

(c) $\text{SUM} (A, B, C) = \sqrt{A + B + C}$ where $A, B, C$ are positive integers. Find the value of $A$ using the given statement function.
SECTION - D

(Quantitative Economics)

13. (a) Define a time-series. Mention its important components with illustrations and describe the method of smoothing data.

(b) Explain the measurement of cyclic movement of a time series and identify the importance of Periodogram Analysis.

(c) Explain the working of variate difference method for trend analysis.

14. (a) Define Laspeyre, Paasche and Marshall-Edgeworth index number and show that M-E index number lies between Laspeyre and Paasche index numbers.

(b) State and explain the criteria of a good index number.

(c) Explain Base shifting, Splicing and Deflating index numbers.

15. (a) Explain heteroscedasticity, its estimation in the case of grouped data and a method of testing for heteroscedasticity.

(b) Formulate a general linear model (GLM) clearly stating the assumptions. Derive the MLE of the parameter vector when the errors in the GLM follow a multivariate normal distribution.

(c) Explain the estimation procedure with autocorrelated disturbances.

16. (a) What is a demand function? Describe normal conditions of demand. If $AR$ and $MR$ denote the average and marginal revenues at any output, then show that the elasticity of demand for the product is given by $\frac{AR}{AR - MR}$.

(b) Mention the main sources that cause serially correlated disturbances. Outline the Cochrane-Orcutt procedure to estimate the parameters of a linear model when the errors are serially correlated.

(c) Compare the ILS and 2 SLS methods of estimation in simultaneous equation models.
13. (a) ಲೋಹ ಜುಗ-ಹಿಗೆಗಳು, ಹಳೊಲಾರೊಯಿತೇ ಭಳ ಸುಮಾರು ವಿಧಾನವು ಇದು ಕರೋ ಅಥವಾ ಮೂಲಕ್ಕೆ ಸಭಾ (Smoothing data) ರೂಪದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(b) ಲೋಹ ಜುಗ-ಹಿಗೆಗಳು ಭಳ ಸುಮಾರು ವಿಧಾನವು ಇದು ಕರೋ ಅಥವಾ ಮೂಲಕ್ಕೆ ಸಭಾ (Periodogram Analysis) ರೂಪದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(c) ವೃತ್ತಿಯ ಮೂಲಕ್ಕೆ ಸಭಾದವರ ಮಹತ್ತೆ (Variate difference method) ರೂಪದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

14. (a) ಸ್ಥಾನತಂತ್ರ, ಅಥವ ಗುಣಗಳು-ಸರಣಿಗಳು ವಿಧಾನವು ಇದು ಮೂಲಕ್ಕೆ M-E ಮಹತ್ತೆಯಲ್ಲಿ ಸ್ಥಾನತಂತ್ರ, ಅಥವ ಗುಣಗಳು-ಸರಣಿಗಳು ವಿಧಾನವು ಮೂಲಕ್ಕೆ ಸಭಾದವರ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(b) ಲೋಹ ಜುಗ-ಹಿಗೆಗಳು ಭಳ ಸುಮಾರು ವಿಧಾನವು ಇದು ಮೂಲಕ್ಕೆ ಸಭಾದವರ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(c) ಸರಣಿ ಸೇಜದಾಯಿ, ಸಭಾದವರ (Splicing) ಮಹತ್ತೆ, ಸಭಾದವರ (Deflating) ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

15. (a) ಸಂಖ್ಯಾ ಸ್ಥಾನವಾತ್ರೆಗಳು, ಹೆಸರಿನ ಮಹತ್ತೆಯಾದ (Heteroscedasticity) ರೂಪದ ಸ್ಥಾನತಂತ್ರದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(b) ಲೋಹ ಜುಗ-ಹಿಗೆಗಳು ಭಳ ಸುಮಾರು ವಿಧಾನ (GLM) ರೂಪದ ಸರಣಿಯ ಹೆಸರಿನ ಮಹತ್ತೆಯಾದ ಸ್ಥಾನತಂತ್ರದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

(c) ಸರಣಿಯ ಮಹತ್ತೆಯಾದ ಸ್ಥಾನತಂತ್ರದ ಮಹತ್ತೆ ಹೊಂದಿದ್ದಾರೆ.

16. (a) ಭಳ ಸುಮಾರು ವಿಧಾನದ (AR) ಮಹತ್ತೆ ಮೂಲಕ್ಕೆ AR, MR ಮಹತ್ತೆಗಳು, ಅರಂಭಗಳು ಸರಣಿಯ ಮಹತ್ತೆಯಾದ (Marginal revenues) ರೂಪದ ಸೇಜದಾಯಿ, AR - MR ಮಹತ್ತೆಯಾದ (Simultaneous equation) ರೂಪದ ಸೇಜದಾಯಿಗಳು 2 SLS ಮಹತ್ತೆಯಾದ ಮಹತ್ತೆ.
SECTION - E

(Demography and Psychrometry)

17. (a) Explain civil registration and population register as sources of demographic data, in particular, of Indian system.

(b) Provide a note on the growth and prospects of world population.

(c) Define the following and explain their uses:

(i) Density and proximity of population

(ii) Infant mortality rate.

18. (a) Describe the measures of mortality based on death statistics.

(b) Stating the underlying assumptions, outline the construction of a life table.

(c) What do you understand by age-structure of a population? Describe the nature of aging of the population.

19. (a) State the nature of models on fertility and human reproduction and describe William Brass Model on fertility.

(b) Mention the kinds of migration and explain the estimation procedure by survival ratio method.

(c) Describe stable and quasi-stable population models. How are these models useful for estimating demographic parameters?
17. (a) 

(b) 

(c) 

(i) 

(ii) 

18. (a) 

(b) 

(c) 

19. (a) 

(b) 

(c) 

(Stable and quasi-stable) 

( Turn over)
20. (a) What is test reliability? How is it measured? Describe any one method in detail.

(b) Explain scaling of rankings or scaling of ratings in terms of normal curve.

(c) Examine the effects of lengthening and repeating a test on test reliability.
(a) ಕನಸುಗಳು ನ್ಯೂಸ್ ಹೆಚ್ಚಾಗಿದ್ದು ಎಬ್ಬರ? ಎಬ್ಬರನ್ನು ಪರಿಹರಿಸುವಾಗ ಆದರೆ ಅದನ್ನು ಎಬ್ಬರದ ಹೆಚ್ಚಾಗಿ ಏತೆ?

(b) ಮುಂದಿನವು ಮುಂದಿನವು ತೆರೆಯಿಕೆಗಳನ್ನು ಸಲ್ಲಿಸಿದರೆ ಸಹಾಯದಲ್ಲಿಯಾಗಿ ಪ್ರಾರಂಭವಾಗಿ ತೆರೆಯಿಕೆಗಳನ್ನು ನೋಡಿ.

(c) ಸಿಹಿಯ ನ್ಯೂಸ್ ಮತ್ತು ಹೆಚ್ಚಾಗಿದ್ದು ಎಬ್ಬರ ಮತ್ತು ಹೆಚ್ಚಾಗಿದ್ದು ಎಬ್ಬರ ತೆರೆಯಿಕೆಗಳನ್ನು ಸಹಾಯಿಸಿ.