

2005
STATISTICS

Paper 1

Time : 3 Hours]

[Maximum Marks : 300

INSTRUCTIONS

*Candidates should attempt **all** the questions in Parts A, B & C. However, they have to choose only **three** questions in Part D. The number of marks carried by each question is indicated at the end of the question.*

Answers must be written in English.

This paper has four parts :

A	20 marks
B	100 marks
C	90 marks
D	90 marks

Marks allotted to each question are indicated in each part.

Assume suitable data if considered necessary and indicate the same clearly.

Notations and symbols used are as usual.

PART A

4×5=20

Each question carries 5 marks.

1. (a) Define probability measure. If $A \subset B$ are two events, then prove that $P(B - A) = P(B) - P(A)$.
- (b) Define unbiased estimator. If u and v are unbiased estimators of θ , then prove that $(au + bv)$ is unbiased for θ , if $a + b = 1$.
- (c) Let $H_0 : X \sim f_0(x) = 1, 0 < x < 1$, and $H_1 : X \sim f_1(x) = 2x, 0 < x < 1$. Based on a single observation obtain most powerful test for H_0 against H_1 , when $\alpha = 0.1$.
- (d) If X and Y are independent standard normal variates then prove that $X + Y$ and $X - Y$ are independent normal variates.

PART B

10×10=100

Each question carries 10 marks.

1. Let (XY) have the density function $f(x, y) = k$, for $0 < x, y$ and $x + y \leq 2$
 - (i) Find k
 - (ii) Obtain the marginal density of X

2. A committee of three members has been formed by selecting the members at random, from a group of 3 Statisticians, 2 Economists and 1 Doctor.
 - (a) Find the probability that the committee consists of 1 Statistician, 1 Economist and 1 Doctor.
 - (b) Find the probability that committee does not consist of an economist.

3. Define characteristic function. (c.f.). Let $P[X = -1] = P[X = 1] = 1/2$. Find c.f. of X and hence find all moments of X .

4. Let $X_1, X_2, \dots, X_n, \dots$ be iid $u(0, \theta)$ random variables and $X_{(n)} = \max(x_1, \dots, x_n)$. Prove that $X_{(n)}$ converges to θ in probability.

5. State Factorization Theorem. Let $x_1 \dots x_n$ be iid random variables with p.d.f $\exp\{-(x - \theta)\}$, for $x > \theta$. Obtain non-trivial sufficient statistic for θ .

6. Let x_1, \dots, x_n be iid random variables having $u(\theta, \theta + 1)$ distribution. Obtain maximum likelihood estimator for θ . Is it unique? Justify.

7. Let x_1, \dots, x_n be iid random variables having $N(\theta, 1)$ distribution. Develop LRT test for testing $\theta = 1$.

8. Describe sign test for location. State the assumptions.

9. Consider the linear model $Y = X\beta + \epsilon$. Obtain least square estimator of β based on n observations.

10. Define $N(\mu, \Sigma)$ distribution. If X has $N(\mu, \Sigma)$ distribution, obtain the distribution of $AX + b$.

PART C

6×15=90

Each question carries 15 marks.

1. State and prove Borel Centeth lemma.
2. Let X have d.f.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ (1+x)/5 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

- (i) Sketch the graph of F.
 - (ii) Find E(X) and V(X).
3. Describe weak and strong laws of large numbers. Let $\{X_n\}$ be a sequence of iid random variables with finite second moment. Examine whether $\{X_n\}$ obeys weak law of large numbers.
 4. State and prove CR inequality.
 5. Describe monotone likelihood ratio property. Obtain UMP test for testing $\theta = 1$ against the alternative $\theta > 1$, where θ is the mean of exponential distribution.
 6. Define multiple correlation coefficient and obtain an expression for the same.

PART D

3×30=90

Answer any **three** of the following questions. Each question carries 30 marks.

1. Define various modes of convergences of a sequence of random variables. Give an example of a sequence of random variables which converges in probability but not almost surely. State and prove a necessary and sufficient condition for almost sure convergence of a sequence of random variables.
2. Describe SPRT. Develop SPRT to test the parameter of Bernoulli distribution. Obtain approximate expressions for OC and ASN functions.
3. Distinguish between parametric and non-parametric procedures. Describe one and two sample(s) problems. Show that one sample Kolmogorov Smirnov statistic is distribution free.
4. State Gauss-Markoff theorem. Describe the model for two way analysis of variance, carry out the complete analysis and give the analysis of variance table.
5. Define students t-statistic and Hotellings T^2 -statistic. Indicate various applications of these statistics in statistical inference.

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Paper 2

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[Maximum Marks : 300

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Answers must be written in **English**.

This paper has four parts :

A	20 marks
B	100 marks
C	90 marks
D	90 marks

Marks allotted to each question **are** indicated in each part.

Assume suitable data if considered **necessary** and indicate the same clearly.

Notations and symbols used are **as usual**.

PART A

Each question carries 5 marks.

1. (a) Prove that in SRSWOR sample mean is unbiased for population mean.
- (b) Explain the term "consumers risk" with a suitable example.
- (c) Give a flow-chart to obtain mean and variance of given n numbers.
- (d) Describe type of demographic data and indicate major sources of demographic data.

PART B

10×10=100

Each question carries 10 marks.

1. Define ratio estimator for the **ratio** of population totals and obtain approximate variance of the estimator.
2. Describe systematic sampling. State **some** advantages of the same.
3. Describe a control chart. Explain a **method** of developing a control chart.
4. Describe CUSUM chart. State its **advantages** over \bar{X} -chart.
5. Define OR. Describe the scope of OR by considering suitable models.
6. What is a queueing system ? **Obtain** expression for steady-state probabilities for the M/M/1 queueing system.
7. Describe a Linear Programming Problem. Solve the following by graphical method.
Maximize $z = 3x + 2y$, where $x \leq 10$, $y \leq 15$ and $x + y \leq 20$.
8. Explain a method to determine seasonal variation in a time series.
9. Discuss a method to find the elasticity of a demand for family budget data.
10. Define (i) Gross Reproduction Rate (GRR), and (ii) Net Reproduction Rate (NRR). Prove or disprove : $NRR \leq GRR$.

PART C

6×15

Each question carries 15 marks.

1. Describe stratified sampling. Obtain the variance of the estimator of the population mean, under Neyman allocation.
2. Explain the terms (i) reliability (ii) failure rate. Obtain the reliability function if the failure rate $r(t) = t$, $t > 0$.
3. Let S_1 and S_2 be respectively a series and a parallel system of two components. Let the life times of the components be independent and exponential with mean 1000 hrs. Find the reliability functions of the two systems.
4. In an inventory model develop an expression for optimum quantity to be ordered when the demand is known. Describe the model and assumptions clearly.
5. Describe components of a time series. Explain moving average method for measuring the trend.
6. Describe a stable population model. Explain how this model can be used to estimate the demographic parameters.

PART D

3×30=90

Answer any **three** of the following questions. Each question carries 30 marks.

1. Explain the terms (i) factorial experiment (ii) confounding. Carry out the analysis of a 2^3 -factorial experiment without confounding and also when ABC is confounded.
2. Describe an acceptance sampling plan and clearly indicate the advantages and disadvantages of such plans. Define and describe the uses of OC and AOQL.
3. Describe an assignment problem and explain this can be viewed as an LPP. Explain a method to solve an assignment problem.
4. Describe the importance and limitations of index numbers. Prove that Fisher's ideal index satisfies the time reversal test.
5. Explain the structure of a complete life table. Describe a method of constructing abridged life table.