INSTRUCTIONS

(i) All questions carry equal marks.
(ii) Unless otherwise stated, figures are assumed to lie on a straight line. Unless stated otherwise, the value of 
\(\pi\) is taken to be 3.1416.
(iii) Noquestionpaper will be allowed to leave the examination room.
(iv) Use of log tables or graph paper is not permitted.

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two sections and printed both in KANNADA and in ENGLISH.

Candidate has to attempt FIVE questions in all.

Question No. 1 and 5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE question from each Section.

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Answer must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-cum-Answer (QCA) Booklet in the space provided. No marks will be given for answers written in a medium other than the authorized one.

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Section-A

1. (a) Every linearly independent subset of a finitely generated vector space \( V(F) \) is either a basis of \( V \) or can be extended to form a basis of \( V \).

(b) Let \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be defined by

\[
T(e_1) = e_2 - e_3, \\
T(e_2) = 2e_1 - e_3, \\
T(e_3) = e_1 - e_2 - e_3.
\]

Find the linear transformation

\[
T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \text{ defined by}
\]

\[
T(e_1) = e_1 - e_2, \\
T(e_2) = 2e_1 - e_3, \\
T(e_3) = e_1 - e_2 + e_3.
\]

Also find the range, null space, rank and nullity of \( T \) given \( \{e_1, e_2, e_3\} \) is the standard basis of \( \mathbb{R}^3 \).

2. (a) State Taylor's theorem and hence deduce Maclaurin's expansion. Expand the function \( y = e^x \log(1-x) \) in powers of \( x \) given \( y(0) = 0 \).
(b) \[ \lim_{x \to 0} \left( \frac{\tan x}{x} \right)^3 \]

Evaluate \[ \lim_{x \to 0} \left( \frac{\tan x}{x} \right)^3 \].

(c) A rectangular box open at the top is to have a volume 32 c.c. Find the dimensions of the box requiring least material for its construction.

3. (a) \[ u = 2xy, \, v = x^2 - y^2, \, x = r \cos \theta, \, y = r \sin \theta \]

\[ f(u, \, v) = -4r \]

(b) If \[ u = 2xy, \, v = x^2 - y^2 \] and \[ x = r \cos \theta, \, y = r \sin \theta \]. Prove that the Jacobian of \( u, v \) with respect to \( r \) and \( \theta \) is

\[ f(u, \, v, \, r, \, \theta) = -4r \]

If \( J \) is the Jacobian of the function \( u \) and \( v \) with respect to \( x \) and \( y \) and \( J' \) is the Jacobian of \( x \) and \( y \) with respect to \( u \) and \( v \), then

\[ J \cdot J' = 1 \quad (\text{provided} \quad J \neq 0) \]
(b) \[ Z = f(x, y) \] for \( n \) homogeneous function of \( x, y \) of degree \( n \), then prove that
\[ \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = nz \quad \forall x, y \text{ in domain of } f \]

Hence prove that \[ \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin(2u) \]

given \[ u = \tan^{-1}\left( \frac{x - y}{\sqrt{x^2 + y^2}} \right) \]

4. (a) \[ \int_0^1 x^2 \left( \log \frac{1}{x} \right)^3 \, dx \]

Define Gamma function.

Evaluate \[ \int_0^1 x^2 \left( \log \frac{1}{x} \right)^3 \, dx \] using Gamma function.

(b) \[ \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = nz \]

Using \( \beta \) and \( \gamma \) functions (Beta, Gamma functions), prove that
\[ \int_0^1 x^2 \, dx = \int_0^\infty \frac{1}{\sqrt{1 + x^4}} \, dx = \frac{\pi}{4\sqrt{2}} \quad \text{given domain} \]
5. (a) **What is principle of Virtual work and its converse for forces acting upon a single particle.**

(b) Two small rings can slide on a smooth parabolic wire with its axis vertical and vertex upwards, being connected by a fine string which passes over a smooth peg at the focus. Show that if the rings can rest in any position in which the string is taut they must be of equal weight.

6. (a) Find the angle between the surfaces \( x^2 + y^2 + z^2 = 9 \) and \( z = x^2 + y^2 - 3 \) at the point \((2, -1, 2)\).

(b) \( \vec{F} = (\sin y + z \cos x) \hat{i} + (x \cos y + \sin z) \hat{j} + (y \cos z + \sin x) \hat{k} \)

Show that \( \vec{F} = \nabla \Phi \) and \( \Phi \) is irational. Find \( \Phi \) such that \( \vec{F} = \nabla \Phi \).

(c) For any vector field \( \vec{F} \), prove that

\[ \text{Curl} \text{ Curl} \vec{F} = \nabla \left( \text{div} \vec{F} \right) - \nabla ^2 \vec{F} \]
(d) Find the expression for divergence of a vector function in orthogonal curvilinear coordinates.

7. (a) A particle starting from rest and executing with simple harmonic motion with period 18 seconds, travels 10 feet in 3 seconds. Find the amplitude, maximum velocity and velocity at the end of 3 seconds.

(b) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A and B be the base angles of the triangle and \( \alpha \) the angle of projection, show that \( \tan \alpha = \tan A + \tan B \).

(c) Show that the greatest angle through which a person can oscillate on a swing with the ropes of which can support twice the persons weight at rest is 120°.

(d) A particle describes the curve \( \theta = \tanh \left( \frac{\theta}{\sqrt{2}} \right) \) under the central force towards the pole. Find the law of force.
8. (a) \textit{(Gauss)} \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \mathbf{F} \cdot d\mathbf{S}.

State and prove gauss divergence theorem.

(b) \int_C (\sin z \, dx - \cos x \, dy + \sin y \, dx)

\text{where } \mathbf{F} = (\sin z, -\cos x, \sin y), 0 \leq x \leq \pi, 0 \leq y \leq 1, z = 3.

Evaluate by stoke's theorem \int_C (\sin z \, dx - \cos x \, dy + \sin y \, dx).

where \(C\) is the boundary of the rectangle \(0 \leq x \leq \pi, 0 \leq y \leq 1, z = 3\).
QCA : 15/II
गणित विषय : Paper-II
MATHEMATICS : Paper-II
2014

Time : 3 hours
Maximum marks : 250

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1. (a) Prove that every finite group is isomorphic to a permutation group.

(b) If $G$ is a group and $N$ is a normal subgroup of $G$, then $G/N$ is a group. Suppose $G$ is a group and $N$ is a normal subgroup of $G$. Let $f: G \rightarrow G/N$ be defined by $f(x) = Nx \forall x \in G$, then prove that $f$ is a homomorphism of $G$ onto $G/N$ and Kernel $f = N$.

2. (a) Prove that the relation of isomorphism in the set of all groups is an equivalence relation.

(b) If $G$ is a group such that $(ab)^m = a^mb^m$ for three consecutive integers $m$ for all $a, b \in G$. Show that $G$ is abelian.
3. \[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \]

Solve the partial differential equation \[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \] given that \[ u(0, y) = 2e^{5x} \] by the method of separation of variables.

4. (a) (i) \[ AB \text{ straight line} \]
(ii) \[ c : x = 1, y = t^2 \text{ circular arc} \]
\[ \int_C xz \, dz \quad A(1, 1) \text{ to } B(2, 4) \text{ evaluate.} \]

Evaluate \[ \int_C xz \, dz \text{ from } A(1, 1) \text{ to } B(2, 4) \]
(i) along the straight line \( AB \)
(ii) along the curve \( c : x = 1, y = t^2 \)

(b) \[ f(Z) = \frac{Z}{(Z+1)(Z+2)} \]
(i) \( Z = 0 \)
(ii) \( Z = 2 \)

Expand \( f(Z) = \frac{Z}{(Z+1)(Z+2)} \) in a Taylor's Series about

(i) \( Z = 0 \)
(ii) \( Z = 2 \)
5. (a) Two equal masses are connected by springs having equal spring constant C, so that masses are free to slide on a frictionless table. The ends of springs are attached with the fixed walls. Using the Lagrangian equation, set up the differential equation of vibrating masses.

(b) Find Lagrange's equation of motion of the bob of a simple pendulum.

6. (a) In a transport office two persons are engaged in checking reservation forms, on an average, the first person checks 57% of the forms, while the second does the remaining. The first person has an error rate of 0.03 and second has an error rate of 0.02. A reservation form is selected at random from the total number of forms checked during a day, and is found to have an error. Find the probability that it was checked

(i) by the first

(ii) by the second person
Assuming that half the population are consumers of rice so that the chance of an individual being a consumer is 1/2. Assuming that the 100 investigators each take 10 individuals to see whether they are consumers how many investigators do you expect to report that three people or less are consumers?

7. (a).failure (against) 12.36% 345.82%

State the properties of Poisson distribution.

(b) sample mean \( \bar{x} \) = 12 hours, sample standard deviation \( \sigma = 3 \) hours, sample size 100. Assuming that the data are normally distributed, what percentage of tube lights are expected to have life

(i) More than 15 hours
(ii) Less than 6 hours
(iii) between 10 and 14 hours
Fit a binomial distribution to the following data

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>28</td>
<td>62</td>
<td>46</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

8. (a) \( Z = \phi(x + ay) + \psi(x - ay) \) form the partial differential equation by eliminating the arbitrary functions \( \phi \) and \( \psi \) from the relation

\[ Z = \phi(x + ay) + \psi(x - ay), \text{ where } 'a' \text{ is a specified constant} \]

(b) \( (x^2 - yz)p + (y^2 - zx)q = z^2 - xy \)

Solve: \( (x^2 - yz)p + (y^2 - zx)q = z^2 - xy \)

(c) \( p' + q^2 - 2px - 2qy + 1 = 0 \)

Solve: \( p^2 + q^2 - 2px - 2qy + 1 = 0 \)