Time : 3 hours | **Maximum Marks : 250**

ಸಮಯ : 3 ಗಂಟೆಗಳು | **ಗರಿಷ್ಠ ಅಂಕಗಳು: 250**

23/I-Statistics 1

SECTION—A / ವಿಭಾಗ—A

1.(a) Define (i) convergence in probability, (ii) convergence almost everywhere.

Let $\{X_n\}$ be a sequence of random variables with $P(X_n \quad 1) \quad \frac{1}{n}$ and

$P(X_n \quad 0) \quad 1 \quad \frac{1}{n}$, Show that $\{ X_n \}$ converges to zero in probability. **[20 Marks]**

1.(b) State and prove Chebyshev's inequality. Indicate how it is useful for identifying convergence in probability of sequence of random variables. **[15 Marks]**

1.(c) Define characteristic function. Prove that characteristic function uniquely determines the distribution function. **[15 Marks]**

2.(a) State and prove Lindeberg-Levy central limit theorem. **[12 Marks]**

2.(b) Let X be a binomial random variable with parameters n and p. As n , $p \quad 0$ such that np . Prove that the limiting binomial distribution is Poisson with parameter . **[13 Marks]**

2.(c) State and prove Markov inequality. **[12 Marks]**

2(d) State and prove Borel Cantelli Lemma. **[13 Marks]**

3.(a) A two dimensional random variable (x, y) has the joint probability distribution function given by

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad y \quad 1 \\ 0, & \text{otherwise} \end{matrix}$$

Find the correlation coefficient between x and y. **[25 Marks]**

3.(b) Calculate from the following data the Fisher's Ideal Index and show that it satisfies. Time Reversal and Factor Reversal Tests.

	Item	A	B	C	D	E
Price	1995	8	2	1	2	1
	2003	20	6	2	5	5
Quantity	1995	50	15	20	10	10
	2003	60	10	25	8	30

[15 Marks]

3.(c) In a co-educational institution, out of 200 students 150 were boys. They took an examination and it was found that 120 passed, 10 girls had failed. Is there any association between sex and success in the examination? **[10 Marks]**

4.(a) State and prove Rao - Blackwell and Lehmann - Sheffe theorem. **[15 Marks]**

4.(b) Define t and F distributions and mention two applications of these distributions. **[10 Marks]**

4.(c) A Sample of 100 dry battery cells tested to find the length of life produced the following results is mean = 12 hours and variance = 9 hours. Assume the data to be normally distributed, what percentage of battery cells are expected to have life

(i) More than 15 hours

(ii) less than 6 hours

(iii) between 10 and 14 hours **[25 Marks]**

SECTION—B / ವಿಭಾಗ—B

5.(a) **Describe Wilcoxon signed rank test, how it is different from sign test, explain. Obtain mean and variance of the Wilcoxon signed rank test under the null hypothesis. Prove that its distribution is symmetric about its mean. State the normal approximation of test statistic when n is large.**

[30 Marks]

5.(b) **An experiment was conducted to determine the soil moisture deficit resulting from varying amounts of residual timber left after cutting trees in a forest. The treatments are treatment 1:no timber left; treatment 2:2000 bd ft left; treatment 3:8000 bd ft left. (Board feet is a particular unit of measurement of timber volume.) The measurements of moisture deficit in soil of three treatments y_1, y_2 and y_3 gives the following information : n_1 5, n_2 n_3 6, $\bar{y}_1$ 1.460, $\bar{y}_2$ 1.597, $\bar{y}_3$ 2.695. Total SS = 7.0516 and Treatment SS = 5.2791. Perform ANOVA test and construct confidence intervals for the treatment differences..**

[20 Marks]

6.(a) **Show that Kolmogorov – Smirnov test is distribution free over the class continuous distribution** **[20 Marks]**

6.(b) **If $E(y_1)$ $_2$ $_1$, $E(y_2)$ $_1$ $_2$ $_3$, $E(y_3)$ $_3$. Examine whether the linear parametric function 3 $_1$ $_2$ $_3$ is estimable? If so, obtain its BLUE. Find the variance of BLUE.** **[20 Marks]**

6.(c) **State and prove Gauss-Markov theorem.** **[10 Marks]**

7.(a) **Explain the run test for randomness of the data.** **[10 Marks]**

7.(b) **The following data gives the life time of bulbs of two different brands**

Brand I	:	80,	100,	90,	110,	125,	130,	70	
Brand II	:	100,	120,	80,	140,	130,	160,	115,	120

Test whether the brands differ with respect to average life. **[20 Marks]**

7.(c) **Consider two samples are x = (1, 5, 7, 9, 15, 17, 21, 23) and y = (2, 6, 10, 12, 18, 20, 26, 28, 32). Apply Wilcoxon- Mann-Whitney test to test the hypothesis $H_0 : F(x)$ $G(x)$ x, where F and G are cdf's of x and y respectively.** **[20 Marks]**

8.(a) Find the multiple linear regression equation of X_1 on X_2 and X_3 from the data relating to three variables given below. **[15 Marks]**

X_1	4	6	7	9	13	15
X_2	15	12	8	6	4	3
X_3	30	24	20	14	10	4

8.(b) In a trivariate distribution $_1$ 2, $_2$ 3, $_3$ 3, r_{12} 0.7, r_{23} 0.5, r_{31} 0.5. Find

(i) $r_{23.1}$

(ii) $R_{1.23}$

(iii) $b_{12.3}$

(iv) $b_{13.2}$

(v) $_{1.23}$ **[15 Marks]**

8.(c) A college entrance examination consisted of 3 tests in Mathematics, English and General Knowledge. To test the ability of the examination to predict performance in a statistics course, data concerning a sample of 200 students were gathered and analysed. Let X_1= grade in Statistics course, X_2 grade in Mathematics course, X_3 grade in English course, X_4 grade in G.K course, The following results were obtained.

$\overline{X_1}$ 75, s_1 10, $\overline{X_2}$ 24, s_2 5, $\overline{X_3}$ 15, s_3 3, $\overline{X_4}$ 36, s_4 6, r_{12} 0.9, r_{13} 0.75, r_{14} 0.8, r_{23} 0.7, r_{24} 0.7, r_{34} 0.85

Find the partial correlation coefficient

(i) $r_{12.34}$

(ii) $r_{13.24}$

(iii) $r_{14.23}$ **[20 Marks]**

Time : 3 hours | **Maximum Marks : 250**

ಸಮಯ : 3 ಗಂಟೆಗಳು | **ಗರಿಷ್ಠ ಅಂಕಗಳು: 250**

23/II-Statistics 2

SECTION—A / ವಿಭಾಗ—A

1.(a) **Explain the meaning of sample and sample design. Briefly discuss some most of the popular designs used in Research.** **[25 Marks]**

1.(b) **The following are the number of km/litre which a test driver with three different type of cars have obtained randomly on different occasions**

Car 1	15	14.5	14.8	14.9		
Car 2	13	12.5	13.6	13.8	14	
Car 3	12.8	13.2	12.7	12.6	12.9	13

Using a 5% level of significance perform a one-way ANOVA to examine the hypothesis that the difference in the average mileage in the three types of cars can be attributed to chance.

[Table value = 3.89] **[25 Marks]**

2.(a) **Distinguish between sampling and non-sampling errors. What is the use of sampling in real life** **[25 Marks]**

2.(b) **Suppose the measurement of the cholesterol content was performed in the three different Laboratories with the data below.**

Food	Laboratory		
	I	II	III
A	3.6	4.1	4.0
B	3.1	3.2	3.9
C	3.2	3.5	3.5
D	3.5	3.8	3.8

Perform a two-way ANOVA using 5% level of significance [Table values 4.76, 5.14

[25 Marks]

3. **A Company tried to study the effect of three price levels [Rs.12=A, Rs.15=B, Rs.18=C] on the sales of its product in a Latin square design by controlling the influence of three types of stores (Small, Medium, Large) and three types of packaging labelled as I, II & III as shown below :**

Store Size	Packaging		
	I	II	III
Small	65 A	50 C	59 B
Medium	55 B	68 A	46 C
Large	52 C	58 B	72 A

Set up an ANOVA table for a 3X3 Latin square design to examine whether the three price levels have an equal effect on sales, at 5% level of significance. Table Value=19.00 **[50 Marks]**

4.(a) **Distinguish between defect and defective. Give some examples of defects for which the C-chart is applicable. How do you calculate control limits for C-Chart?** **[15 Marks]**

4.(b) **Describe single sampling plan. Obtain OC and AOQ curve for this plan. Distinguish clearly between AQL (Acceptable quality level) and LTPD (Lot Tolerance Proportion or Percentage Defective).** **[20 Marks]**

4.(c) **Derive the failure rate for Weibull distribution with two parameters. Discuss the behaviour of the curve of failure rate.** **[15 Marks]**

SECTION—B / ವಿಭಾಗ—B

5.(a) Describe an M/M/1 queuing system. Derive the steady state probability distribution of system size of this queuing system. [25 Marks]

5.(b) Solve the following L.P.P by Charnes-M method: [15 Marks]

Max $\quad Z \quad 4x_1 \quad 3x_2$

s.t.

$$2x_1 \quad x_2 \quad 10$$
$$3x_1 \quad 2x_2 \quad 6$$
$$x_1 \quad x_2 \quad 6$$
$$x_1, x_2 \quad 0$$

5.(c) What is an assignment problem? Give an example. Model the assignment problem as a L.P.P. [10 Marks]

6.(a) What is Time Series analysis? Explain Semi-Average method, their advantages and disadvantages in computing a straight line trend as compared with the Least Square method. [25 Marks]

6.(b) What do you mean by Consumers Equilibrium? And, explain the methods to maximise utility function subject to budget constraint. [25 Marks]

7. What is identification? Discuss the types of identification problem. How would you identify the following equations in simultaneous equations system by using order and rank condition rules : [50 Marks]

$$y_1 \quad 3y_2 \quad 2x_1 \quad x_2 \quad u_1$$
$$y_2 \quad y_3 \quad x_3 \quad u_2$$
$$y_3 \quad y_1 \quad y_2 \quad 2x_3 \quad u_3$$

8.(a) What is life table? Mention the uses of life tables. [10 Marks]

8.(b) Explain the construction of life table in terms of its components. [20 Marks]

8.(c) A life table with two years age returns with certain missing values is presented below :

Age (years)	l_x	d_x	P_x	q_x	L_x	T_x	e_x^0	
35	9,345	–	–	–	–	1,63,819	–	
36	9,243	149	–	–	–	–	–	

Complete the life table by filling the blanks. [20 Marks]